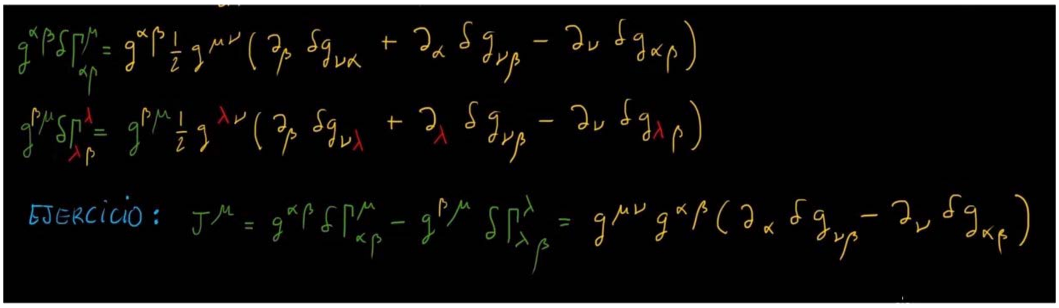

Figure 1:

EJERCICIO Cap. 57 Curso Relatividad General (Javier Gárcia)

$g^{\alpha\beta}\delta\Gamma^{\mu}_{\alpha\beta} = g^{\alpha\beta}1/2g^{\mu\nu}\partial_\beta\delta g_{\nu\alpha} + g^{\alpha\beta}1/2g^{\mu\nu}\partial_\alpha\delta g_{\nu\beta} - g^{\alpha\beta}1/2g^{\mu\nu}\partial_\nu\delta g_{\alpha\beta}$
$g^{\beta\mu}\delta\Gamma^{\lambda}_{\lambda\beta} = g^{\beta\mu}1/2g^{\lambda\nu}\partial_\beta\delta g_{\nu\lambda} + g^{\beta\mu}1/2g^{\lambda\nu}\partial_\lambda\delta g_{\nu\beta} - g^{\beta\mu}1/2g^{\lambda\nu}\partial_\nu\delta g_{\lambda\beta}$
$J^\mu = \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\beta\delta g_{\nu\alpha} + \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\alpha\delta g_{\nu\beta} - \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\nu\delta g_{\alpha\beta} - \frac{1}{2}g^{\beta\mu}g^{\lambda\nu}\partial_\beta\delta g_{\nu\lambda} - \frac{1}{2}g^{\beta\mu}g^{\lambda\nu}\partial_\lambda\delta g_{\nu\beta} + \frac{1}{2}g^{\beta\mu}g^{\lambda\nu}\partial_\nu\delta g_{\lambda\beta})$
En los tres últimos términos sustituimos $\nu \to \beta \;\; ; \;\; \beta \to \nu \;\; ; \;\; \lambda \to \alpha$

En $\frac{1}{2}g^{\beta\mu}g^{\lambda\nu}\partial_\beta\delta g_{\lambda\nu}$ queda $\frac{1}{2}g^{\nu\mu}g^{\alpha\beta}\partial_\nu\delta g_{\alpha\beta} = \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\nu\delta g_{\alpha\beta}$
En $\frac{1}{2}g^{\beta\mu}g^{\lambda\nu}\partial_\lambda\delta g_{\nu\beta}$ queda $\frac{1}{2}g^{\nu\mu}g^{\alpha\beta}\partial_\alpha\delta g_{\nu\beta} = \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\alpha\delta g_{\nu\beta}$
Y en $\frac{1}{2}g^{\beta\mu}g^{\lambda\nu}\delta\partial_\nu g_{\beta\lambda}$ queda $\frac{1}{2}g^{\nu\mu}g^{\alpha\beta}\delta\partial_\beta g_{\nu\alpha} = \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\delta\partial_\beta g_{\alpha\nu}$

Sustituyéndolos
$\frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\beta\delta g_{\nu\alpha} + \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\alpha\delta g_{\nu\beta} - \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\nu\delta g_{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\nu\delta g_{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\partial_\alpha\delta g_{\nu\beta} + \frac{1}{2}g^{\alpha\beta}g^{\mu\nu}\delta\partial_\beta g_{\alpha\nu} =$

$$J^\mu = g^{\alpha\beta}g^{\mu\nu}\partial_\beta\delta g_{\nu\alpha} - g^{\alpha\beta}g^{\mu\nu}\partial_\nu\delta g_{\alpha\beta}$$

Valdría este resultado pero para que coincida exactamente con la solución propuesta basta cambiar en el primer término $\alpha \to \beta \;\; ; \;\; \beta \to \alpha$
$J^\mu = g^{\beta\alpha}g^{\mu\nu}\partial_\alpha\delta g_{\nu\beta} - g^{\alpha\beta}g^{\mu\nu}\partial_\nu\delta g_{\alpha\beta}$

$$J^\mu = g^{\alpha\beta}\delta\Gamma^{\mu}_{\alpha\beta} - g^{\beta\mu}\delta\Gamma^{\lambda}_{\lambda\beta} = g^{\alpha\beta}g^{\mu\nu}(\partial_\alpha\delta g_{\nu\beta} - \partial_\nu\delta g_{\alpha\beta})$$

Ceuta, 9 de julio de 2019
Antonio Gros